



On domination type invariants of regular dendrimer

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Abstract. In this paper domination number, vertex-edge domination number and edge-vertex domination are calculated for regular dendrimers.

Keywords. domination, vertex-edge domination, edge-vertex domination.

1 Introduction

Let $G = (V, E)$ be a simple connected graph with vertex set V and the edge set E . For the open neighborhood of a vertex v in a graph G , the notation $N_G(v)$ is used as $N_G(v) = \{u | (u, v) \in E(G)\}$ and the closed neighborhood of v is used as $N_G[v] = N_G(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighborhood of S is $N(S) = \bigcup_{v \in S} N(v)$ and the closed neighborhood of S is $N[S] = N(S) \cup S$.

A subset $S \subseteq V$ is a dominating set, if every vertex in G either is element of S or is adjacent to at least one vertex in S . The domination number of a graph G denoted by $\gamma(G)$ is equal to the minimum cardinality of a dominating set in G . Fundamental notions of domination theory are outlined in the book [1].

A vertex v ve -dominates an edge e which is incident to v , as well as every edge adjacent to e . A set $S \subseteq V$ is a ve -dominating set if every edges of a graph G is ve -dominated by at least one vertex of S [2, 5]. The minimum cardinality of a ve -dominating set is named ve -domination number denoted by $\gamma_{ve}(G)$.

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An edge e ev -dominates a vertex v which is a vertex of e , as well as every vertex adjacent to v [2, 5]. A subset $D \subseteq E$ is an edge-vertex dominating set (in simply, ev -dominating set) of G , if every vertex of a graph G is ev -dominated by at least one edge of D . The minimum cardinality of a ev -dominating set is named ev -domination number denoted by $\gamma_{ev}(G)$.

We attain three domination type invariants for regular dendrimers.

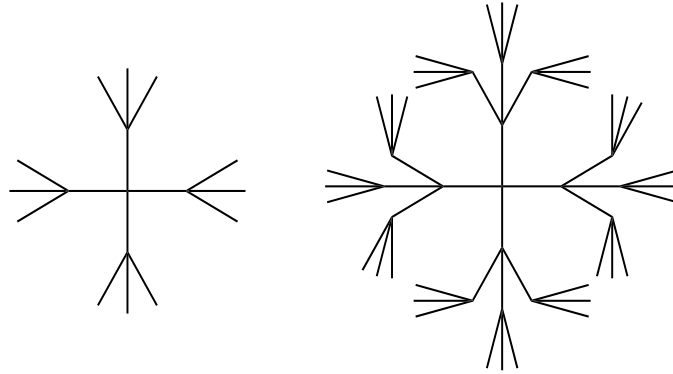


Figure 1. Dendrimers $T_{2,4}$ and $T_{3,4}$

Dendrimers are highly branched trees [4]. A regular dendrimer $T_{k,d}$ is a tree with a central vertex v . Every non-pendant vertex of $T_{k,d}$ is of degree $d \geq 2$ and the radius is k which is the distance from v to each pendant vertex. Dendrimers $T_{2,4}$ and $T_{3,4}$ are demonstrated in Figure 1. Some properties of regular dendrimers are denoted in the following lemma [3].

Lemma 1.1. *If $T_{k,d}$ is a tree with central vertex v , then*

- i) *The order of $T_{k,d}$ is $1 + \frac{d[(d-1)^k - 1]}{d-2}$,*
- ii) *$T_{k,d}$ has d branches,*
- iii) *Each branch of $T_{k,d}$ has $\frac{(d-1)^k - 1}{d-2}$ vertices,*
- iv) *Each branch of $T_{k,d}$ has $(d-1)^{k-1}$ pendant vertices,*
- v) *Each branch of $T_{k,d}$ has $\frac{(d-1)^{k-1} - 1}{d-2}$ nonpendant vertices,*
- vi) *The number of vertices on radius k is $d(d-1)^{k-1}$.*

2 Main Results

We remind some well known properties of paths and cycles in the next lemma.

Lemma 2.1. Let P_n be a path and C_n a cycle with n vertices [5],

$$i) \quad \gamma(P_n) = \gamma(C_n) = \lceil \frac{n}{3} \rceil$$

$$ii) \quad \gamma_{ve}(P_n) = \gamma_{ev}(P_n) = \lfloor \frac{n+2}{4} \rfloor$$

$$iii) \quad \gamma_{ve}(C_n) = \gamma_{ev}(C_n) = \lfloor \frac{n+3}{4} \rfloor$$

Theorem 2.2. If $T_{k,d}$ is a regular dendrimer, then

$$\gamma(T_{k,d}) = \begin{cases} 1 + \frac{(d-1)^k - d + 1}{d-2}, & k \text{ is odd} \\ \frac{(d-1)^k - 1}{d-2}, & k \text{ is even} \end{cases}$$

Proof. Let k be odd. In this case the minimum cardinality dominating set of $T_{k,d}$ is consisted of central vertex v , vertices on radius $k = 2, 4, \dots, k - 1$. Summation of all vertices is by Lemma 1.1 (vi)

$$\gamma(T_{k,d}) = 1 + d(d-1) + d(d-1)^3 + \dots + d(d-1)^{k-2}$$

$$\gamma(T_{k,d}) = 1 + d(d-1) \left[1 + (d-1)^2 + \dots + (d-1)^{k-3} \right]$$

The second term of this equation is a geometric series such that $r = (d-1)^2$. So,

$$\gamma(T_{k,d}) = 1 + d(d-1) \frac{r^{\frac{k-3}{2}+1} - 1}{r-1}$$

$$\gamma(T_{k,d}) = 1 + d(d-1) \frac{r^{\frac{k-1}{2}} - 1}{r-1}$$

$$\gamma(T_{k,d}) = 1 + d(d-1) \frac{(d-1)^{k-1} - 1}{(d-1)^2 - 1}$$

$$\gamma(T_{k,d}) = 1 + \frac{(d-1)^k - d + 1}{d-2}$$

Now let k is even. In this case the minimum dominating set of $T_{k,d}$ is consisted of vertices on radius k , where $k = 1, 3, \dots, k - 1$. Therefore

$$\gamma(T_{k,d}) = d + d(d-1)^2 + d(d-1)^4 + \dots + d(d-1)^{k-2}$$

$$\gamma(T_{k,d}) = d \left[1 + (d-1)^2 + (d-1)^4 + \dots + (d-1)^{k-2} \right].$$

This equation is a geometric series such that $r = (d-1)^2$ and then

$$\gamma(T_{k,d}) = d \frac{r^{\frac{k-2}{2}+1} - 1}{r-1}$$

$$\gamma(T_{k,d}) = d \frac{r^{\frac{k}{2}} - 1}{r - 1}$$

$$\gamma(T_{k,d}) = d \frac{(d - 1)^k - 1}{(d - 1)^2 - 1}$$

$$\gamma(T_{k,d}) = \frac{(d - 1)^k - 1}{d - 2}.$$

□

Theorem 2.3. *If $T_{k,d}$ is a regular dendrimer, then*

$$\gamma_{ve}(T_{k,d}) = \begin{cases} 1 + \frac{(d - 1)^{k-1} - d + 1}{d - 2}, & k \text{ is even} \\ \frac{(d - 1)^{k-1} - 1}{d - 2}, & k \text{ is odd} \end{cases}$$

Proof. A vertex v ve -dominates an edge e which is incident to v , as well as every edge adjacent to e . This means a vertex ve -dominates every edge exist in maximum distance 2 from it. By this way we assume that k is even. In this case minimum ve -dominating set of $T_{k,d}$ is consisted of central vertex v , vertices on radius $k = 2, 4, \dots, k - 2$. Thus

$$\begin{aligned} \gamma_{ve}(T_{k,d}) &= 1 + d(d - 1) + d(d - 1)^3 + \dots + d(d - 1)^{k-3} \\ \gamma_{ve}(T_{k,d}) &= 1 + d(d - 1) \left[1 + (d - 1)^2 + \dots + (d - 1)^{k-4} \right] \end{aligned}$$

The second term of this equation is a geometric series such that $r = (d - 1)^2$. So,

$$\begin{aligned} \gamma(T_{k,d}) &= 1 + d(d - 1) \frac{r^{\frac{k-4}{2}+1} - 1}{r - 1} \\ \gamma(T_{k,d}) &= 1 + d(d - 1) \frac{r^{\frac{k-2}{2}} - 1}{r - 1} \\ \gamma(T_{k,d}) &= 1 + d(d - 1) \frac{(d - 1)^{k-2} - 1}{(d - 1)^2 - 1} \\ \gamma(T_{k,d}) &= 1 + \frac{(d - 1)^{k-1} - d + 1}{d - 2} \end{aligned}$$

Now let k is odd. In this case ve -dominating set of $T_{k,d}$ is consisted of vertices on radius $k = 1, 3, \dots, k - 2$. Therefore

$$\begin{aligned} \gamma_{ve}(T_{k,d}) &= d + d(d - 1)^2 + d(d - 1)^4 + \dots + d(d - 1)^{k-3} \\ \gamma_{ve}(T_{k,d}) &= d \left[1 + (d - 1)^2 + (d - 1)^4 + \dots + (d - 1)^{k-3} \right]. \end{aligned}$$

This equation is a geometric series such that $r = (d - 1)^2$ and then

$$\begin{aligned}\gamma_{ve}(T_{k,d}) &= d \frac{r^{\frac{k-3}{2}+1} - 1}{r - 1} \\ \gamma_{ve}(T_{k,d}) &= d \frac{r^{\frac{k-1}{2}} - 1}{r - 1} \\ \gamma_{ve}(T_{k,d}) &= d \frac{(d - 1)^{k-1} - 1}{(d - 1)^2 - 1} \\ \gamma_{ve}(T_{k,d}) &= \frac{(d - 1)^{k-1} - 1}{d - 2}.\end{aligned}$$

□

Theorem 2.4. *If $T_{k,d}$ is a regular dendrimer, then*

$$\gamma_{ve}(T_{k,d}) = \gamma_{ev}(T_{k,d}).$$

Proof. We investigate $T_{1,d}$ firstly. The minimum cardinality ev -dominating set of $T_{1,d}$ is consisted of one of three edges which is incident the central vertex v . If we take $k = 3$ the minimum cardinality ev -dominating set of $T_{3,d}$ is consisted of edges lying between $k = 1$ and $k = 2$. The number of these edges are equal to the pendant vertices of $T_{2,d}$. If we continue this method, ev -domination number of $T_{k,d}$ is equal to the ve -domination number of $T_{k,d}$ when k is odd.

For the $T_{2,d}$ the minimum cardinality ev -dominating set of $T_{2,d}$ is consisted three edges incident the central vertex v . For the $T_{4,d}$ the minimum cardinality ev -dominating set is consisted of edges which are incident the v and the edges lying between $k = 2$ and $k = 3$. Number of the second type vertices is equal to the number of pendant vertices of $T_{3,d}$. If we continue like this, ev -domination number of $T_{k,d}$ is equal to the ve -domination number of $T_{k,d}$ when k even.

□

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